2022 Fall PHYS2310 電磁學（Electromagnetism) Midterm (double sides)

[Griffiths Chs. 1-3] 2022/11/15, 10:10am – 12:00am, 教師：張存績

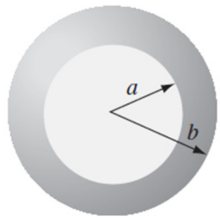
◇ Useful formulas $\nabla V = \frac{\partial V}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\boldsymbol{\phi}}$

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}v_\phi$$

$$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right)\hat{\boldsymbol{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}}.$$

1. (a) Find the divergence of the function $\mathbf{v} = (r\cos\theta)\,\hat{\mathbf{r}} + (r\sin\theta)\hat{\boldsymbol{\theta}} + (r\sin\theta\cos\phi)\hat{\boldsymbol{\phi}}$. (7%)
(b) Test the divergence theorem for this function, using as your volume the inverted hemispherical bowl of radius R, resting on the xy plane and center at the origin. (7%)
(c) Find the curl of **v**. (6%)

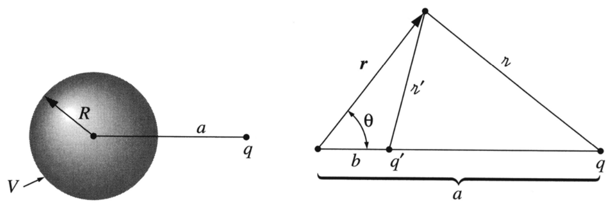
2. Consider a thick spherical shell with the charge density:



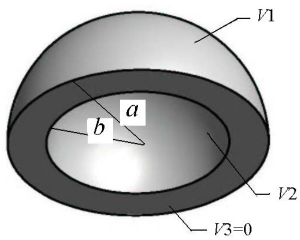
$$\rho = \begin{cases} \dfrac{\rho_0 r}{a}, & r < a \\ \dfrac{\rho_0 a^2}{r^2}, & a < r < b \end{cases}$$

and a perfect conducting shell with surface charge density σ_b is placed at $r = b$, making $\mathbf{E} = 0$ for $r > b$. Find the electric field in the three regions:
(a) $r < a$, (5%)
(b) $a < r < b$, (5%)
(c) $r > b$, (5%)
(d) Find the value of σ_b. (5%)

3. A point charge q is situated at distance a from the center of a conducting sphere of radius R. The sphere is maintained at the constant potential V.
(a) If V=0, find the position and value of the image charge. (5%)
(b) If V=V_0, find the potential outside the sphere. (5%)
(c) Find the electric field on the surface of the metal sphere. (5%)
(d) Find the surface charge density and the total charge on the metal sphere. (5%)
[Hint: 1. use the notations shown below. 2. Assume q lays on the z-axis]



4. Suppose the potential on the surface of a hollow hemisphere is specified, as shown in the figure, where $V_1(a,\theta)=V_0(5\cos^3\theta-3\cos\theta)$, $V_2(b,\theta)=0$, $V_3(r,\pi/2)=0$. V_0 is a constant.



(a) Show the general solution in the region $b \le r \le a$. (4%)

(b) Determine the potential in the region $b \le r \le a$, using the boundary conditions. (10%)

(c) Calculate the electric field on the surface of the outer shell $\mathbf{E}(r=a)$. (6%)

[Hint: $P_0(x)=1$, $P_1(x)=x$, $P_2(x)=(3x^2-1)/2$, and $P_3(x)=(5x^3-3x)/2$.]

5. An idea electric dipole **p** is situated at the origin, and points in the *z* direction. An electric charge *q*, of mass *m*, is released from rest at a point in the *xy* plane. The potential of the dipole is $V(\mathbf{r})=(1/4\pi\varepsilon_0)(p\cos\theta/r^2)$ and the gravitational force points in the –*z* direction.

(a) Find the electric force between the dipole and the charge. (8%)

(b) Find the total force (electrical and gravitational) on the charge. (6%)

(c) Find the total potential energy. (6%)